Variable thermal conductivity on Jeffery fluid past a vertical porous plate with heat and mass fluxes

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ABSTRACT

In this study, the influence of variable thermal conductivity on Jeffery fluid with heat and mass transfer was studied. The Rosseland approximation has been used to describe the radiative heat flux in the energy equation. The governing equations in dimensionless form have been solved numerically by implicit finite difference schemes of Crank-Nicolson type. Results were presented graphically showing the effects of various physical parameters associated with the fluid such as; Jeffery parameter, thermal Grashof number, solutal Grashof number, Schmidt number, Prandtl number, magnetic parameter, Dufour number, permeability parameter, suction parameter, heat generation parameter, and Eckert number on the velocity, temperature and concentration.

Keywords: Jeffery fluid; porous medium; Radiation; Magnetohydrodynamic (MHD), Heat and mass transfer.

1.0 INTRODUCTION

The study of flow of electrically conducting fluid past a porous plate is given primary importance due to its numerous applications in Science and Engineering field like magnetohydrodynamics (MHD) generators, nuclear reactors, oil exploration, boundary layer control in the field of aerodynamics, plasma studies, geothermal energy extraction, solar collectors, jet nozzles, turbine blades and heat exchangers. Specifically, they are greatly used to insulate a heated body to maintain its temperature.

Effects of a magnetic field on the free convective flow through porous medium bounded by an infinite vertical plate with constant heat flux was investigated by Raptis (1983). Abel and Mahesha (2008) studied heat and mass transfer on MHD viscoelastic fluid over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. Bisht et al. (2011) examined the steady incompressible mixed convection boundary layer flow with variable fluid properties and mass transfer inside a cone to point sink at the vertex of the cone numerically. Chaim (1998) analysed heat transfer in a fluid with variable thermal conductivity over a stretching sheet.

Nadeem et al. (2011) investigated the boundary layer flow of a Jeffery fluid over exponentially stretching surface. The effect of thermal radiation was carried out for two cases. The reduced similarity equations were solved by homotopy analysis method (HAM).
Nasrin and Alim (2010) examined the effects of variable thermal conductivity on the coupling of conduction and joule heating with MHD free convection flow along vertical plate. Effects of radiation and thermal diffusivity on heat transfer over a stretching surface with variable heat flux was carried out by Seddeek and Abelmeguid (2006). Seddeek et al. (2009) conducted numerical study for the thermophoresis and variable thermal conductivity on heat and mass transfer over an accelerated surface with heat source.

Das et al. (1994) considered the effects of first order chemical reaction on the flow past an impulsively stated vertical plate with constant heat flux and mass transfer, Muthucumaraswamy and Ganesan (2001) and Muthcumaraswamy (2002) studied first order homogenous chemical reaction flow past an infinite plate. Sudhakar et al. (2012) researched chemical reaction effects on MHD free convection flow past an infinite vertical accelerated plate with constant heat flux, thermal diffusion and diffusion thermo.


Hamad et al. (2013) analysed numerically using finite difference method jump effects on boundary layer flow of a Jeffery fluid near stagnation point on stretching/shrinking sheet with variable thermal conductivity. They investigated how the flow field, temperature field, shear stress, and heat flux vary within the boundary layer with thermal jump at the wall when the thermal conductivity is temperature dependent. Uwanta and Omokhuale (2014) investigated effects of variable thermal conductivity on heat and mass transfer with Jeffery fluid numerically using implicit finite difference method. Results were shown graphically for the effect of parameters associated with the flow. Idowu et al. (2013) reported effect of heat and mass transfer on unsteady MHD oscillatory flow of Jeffery fluid in a horizontal channel with chemical reaction analytically using perturbation method. The graphs and tables were used to discuss in details the effects of various emerging parameters on the velocity, temperature and concentration.

The aim of this study is to examine the influence of variable thermal conductivity on Jeffery fluid past a vertical porous plate with heat and mass fluxes. The governing dimensionless partial differential equations (PDEs) have been solved numerically using implicit finite difference schemes of Crank-Nicolson type. The results obtained show that the velocity field rise as $R$, $Gr$, $Gc$, $Du$, $Sr$, $S$, $\eta$, $t$ and $\lambda_1$ increased but reduces for $Pr$, $Sc$, $M$, $N$, $K$, and $\gamma$. The temperature profile increases due to the presence of radiation, heat generation, Dufour number and time but decreases for increased values of Prandtl number and suction. Similarly,
concentration rises with Soret number and time, and falls with increasing values of Schmidt number Sc, chemical reaction parameter and suction.

2.0 PROBLEM FORMULATION

An unsteady two–dimensional heat and mass transfer flow of an incompressible electrically conducting viscous fluid past an infinite vertical porous plate moving with Jeffery fluid is examined. The x-axis is taken on an infinite plate, and parallel to the free steam velocity which is vertical and the y-axis is taken normal to the plate. A magnetic field $B_0$ of uniform strength is applied transversely to the direction of the flow. Where fluid suction or injection and magnetic field are imposed at the plate surface. The temperature and concentration of the fluid are raised to $T'_w$ and $C'_w$ respectively and are higher than the ambient temperature and that of fluid. In addition, the Soret, radiation, chemical reaction and variable thermal conductivity effect is taken into account. It is assumed that induced magnetic field is negligible, viscous dissipation and the heat generated are not neglected.

Assuming the Boussinesq and boundary-layer approximations hold, the basic equations which govern the problem are:

$$\frac{\partial v'}{\partial y'} = 0$$

$$\frac{\partial u'}{\partial t'} + v \frac{\partial u'}{\partial y'} = \frac{v}{1+\lambda_1} \frac{\partial^2 u'}{\partial y'^2} + g \beta (T'_w - T'_\infty) + g \beta' (C'_w - C'_\infty) - \frac{\sigma B_0^2}{\rho} u' - \frac{v}{K} u'$$

$$\frac{\partial T'}{\partial t'} + v \frac{\partial T'}{\partial y'} = \frac{k}{\rho Cp} \frac{\partial}{\partial y'} \left[ K_T \frac{\partial T'}{\partial y'} \right] + \frac{Q}{\rho Cp} (T'_w - T'_\infty)$$

$$+ \frac{D_m k_T}{C_s \rho Cp} \frac{\partial^2 C'}{\partial y'^2} - \frac{1}{\rho Cp} \frac{\partial q_r}{\partial y'} + \frac{\mu}{Cp} \left( \frac{\partial u'}{\partial y'} \right)^2$$

$$\frac{\partial C'}{\partial t'} + v \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} + \frac{D_m k_T}{T_s} \frac{\partial^2 T'}{\partial y'^2} - R' (C'_w - C'_\infty)$$

with the following initial and boundary conditions:

$$t' \leq 0, u' = 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ for all } y'$$

$$t' > 0, \frac{\partial T'}{\partial y'} = -\frac{q}{k}, \frac{\partial C'}{\partial y'} = -\frac{q}{k} \text{ at } y' = 0$$

$$u' \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty$$

The radiative heat flux term by using Rosseland approximation is given by

$$q_r = -\frac{4\sigma^4 T^4}{3a_R}$$
where \( u \) and \( v \) are velocity components in \( x' \) and \( y' \) directions respectively, \( T \) is the temperature, \( t \) is the time, \( g \) is the acceleration due to gravity, \( \beta \) is the thermal expansion coefficient, \( \beta^* \) is the concentration expansion coefficient, \( \nu \) is the kinematic viscosity, \( D \) is the chemical molecular diffusivity, \( D_e \) is the coefficient of temperature diffusivity, \( C_p \) is heat capacity at constant pressure, \( B_o \) is a constant magnetic field intensity, \( \sigma \) is the electrical conductivity of the fluid, \( q_r \) is the radiative heat flux, \( \sigma^* \) is the Stefan-Boltzmann constant, \( k \) is the variable thermal conductivity, \( C_s \) is the concentration susceptibility, \( \rho \) is the density, \( \lambda \) is the Jeffery fluid, \( q \) is the constant heat flux per unit area at the plate, \( T_s \) is the mean fluid temperature \( T_w \) is the wall temperature, \( T_\infty \) is the free stream temperature, \( C_w \) is the species concentration at the plate surface, \( C_{\infty} \) is the free stream concentration, \( Q \) is the heat generation coefficient. \( v_0 > 0 \) is the suction parameter and \( v_0 < 0 \) is the injection parameter. On introducing the following non-dimensionless quantities

\[
\theta = \frac{u'}{u_0}, \quad \gamma = \frac{v' u_0}{\nu}, \quad t = \frac{t' u_0}{t_0}, \quad \theta = \frac{(T' - T_\infty)}{q' v}, \quad C = \frac{(C'_s - C'_\infty)}{q' v}
\]

(7)

where \( u_0 \) and \( t_0 \) are reference velocity and time respectively.

Using (1) and (7), equations (2) - (4) are transformed to the following:

\[
\frac{\partial u}{\partial t} - \gamma \frac{\partial u}{\partial y} = \frac{1}{1+\lambda_1} \frac{\partial^2 u}{\partial y^2} + Gr \theta + GcC - Mu - Ku \quad (8)
\]

\[
\frac{\partial \theta}{\partial t} - \gamma \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left[ 1 + \frac{4}{3} R + \eta \theta \right] \frac{\partial^2 \theta}{\partial y^2} + \eta \frac{\partial \theta}{\partial y} + S \theta + Du \frac{\partial^2 C}{\partial y^2} + Ec \left( \frac{\partial u}{\partial y} \right)^2 \quad (9)
\]

\[
\frac{\partial C}{\partial t} - \gamma \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} - RC \quad (10)
\]

The corresponding boundary conditions are:

\[
\begin{align*}
&u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } y : t \leq 0 \\
&u = 0, \quad \frac{\partial \theta}{\partial y} = -1, \quad \frac{\partial C}{\partial y} = -1 \quad \text{at } y = 0 \\
&u = 0, \quad \theta = 0, \quad C = 0 \quad \text{as } y \to \infty
\end{align*}
\]

(11)
where $Gr$ is the thermal Grashof number, $Gc$ is the mass Grashof number, $Sc$ is the Schmidt number, $Pr$ is the Prandtl number, $M$ is the magnetic parameter, $Du$ is the Dufour number, $Sr$ is the Soret number, $K$ is the permeability parameter, $\gamma$ is the suction parameter, $N$ is chemical reaction parameter, $R$ is radiation, $S$ is the heat generation parameter, $Ec$ is the Eckert number. $\eta$ is a constant.

Equations (7) to (11) are now solved by implicit finite difference schemes of Crank – Nicolson type. The finite difference approximations of these equations are as follows:

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t} - \gamma \frac{u_{i+1,j}-u_{i,j}}{\Delta y} = \frac{1}{(1+\lambda)} \left[ \frac{u_{i+1,j}+u_{i-1,j}-2u_{i,j}+u_{i+1,j+1}+u_{i-1,j+1}-2u_{i,j+1}}{2(\Delta y)^2} \right]$$

$$+ \frac{Gr}{2} \left( \frac{\theta_{i,j+1}+\theta_{i,j}}{\Delta y} \right) + \frac{Gc}{2} \left( C_{i,j+1}+C_{i,j} \right) - \frac{M}{2} \left( u_{i,j+1}+u_{i,j} \right) - \frac{K}{2} \left( u_{i,j+1}+u_{i,j} \right)$$

(12)

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t} - \gamma \frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y} = \frac{1}{Pr} \left[ 1 + \frac{4}{3} R + \frac{\eta}{2} \left( \theta_{i,j+1}+\theta_{i,j} \right) \right] \left[ \frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j+1}+\theta_{i-1,j+1}-2\theta_{i,j+1}}{2(\Delta y)^2} \right]$$

$$+ \frac{\eta}{Pr} \left( \frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y} \right) + \frac{S}{2} \left( \theta_{i,j+1}+\theta_{i,j} \right) + \frac{Du}{2} \left( \theta_{i,j+1}+\theta_{i,j} \right) + Ec \left( \frac{u_{i+1,j}-u_{i,j}}{\Delta y} \right)^2$$

(13)

$$\frac{C_{i,j+1}-C_{i,j}}{\Delta t} - \gamma \frac{C_{i+1,j}-C_{i,j}}{\Delta y} = \frac{1}{Sc} \left[ \frac{C_{i+1,j}-C_{i-1,j}-2C_{i,j}+C_{i+1,j+1}+C_{i-1,j+1}-2C_{i,j+1}}{2(\Delta y)^2} \right]$$

$$+ \frac{Sr}{2} \left[ \frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j+1}+\theta_{i-1,j+1}-2\theta_{i,j+1}}{\Delta y} \right] = \frac{N}{2} \left( C_{i,j+1}+C_{i,j} \right)$$

(14)

The initial and boundary conditions become

$$u_{i,0} = 0, \theta = 0, C_{i,0} = 0 \text{ for all } i \text{ except } i = 0$$

$$\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y} = -1, \frac{C_{i+1,j}-C_{i,j}}{\Delta y} = -1$$

(15)

$$u_{t,0} = 0, \theta_{t,0} = 0, C_{t,0} = 0$$

where $l$ corresponds to $\infty$. The suffix i corresponds to $y$ and j is equals to $t$. consequently, $\Delta t = t_{j+1} - t_j$ and $\Delta y = y_{i+1} - y_i$.

### 3.0 NUMERICAL PROCEDURE

In order to access the effects of parameters on the flow variables namely; Jeffery parameter, thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, magnetic parameter, Soret number, Dufour number, permeability parameter, suction parameter, heat generation parameter, radiation parameter and Eckert number on the velocity, temperature and concentration, and have grips of the physical problem, the unsteady coupled non-linear partial differential equations (8) – (10) with boundary conditions (11) have been solved by
implicit finite difference schemes of Crank – Nicolson type. This method converges fast and is unconditionally stable. The finite difference approximations of these equations were solved by using the values for $Gr = Gc = M = \eta = S = R = Sr = 1$, $\lambda_i = 0.5$, $Pr = 0.71$, $Sc = 0.3$, $Ec = 0.2$, $Du = 0.03$, $N = 0.5$, $K= 0.5$, $\gamma = 0.5$ except where they are varied. A step size of $\Delta Y = 0.01$ is used for the interval $Y_{\min} = 0$ to $Y_{\max} = 5$ for a desired accuracy and a convergence criterion of $10^{-6}$ is satisfied for various parameters.

5.0 RESULTS AND DISCUSSION

Knowing the values of $C$, $\theta$, $u$ at time $t$, the values at a time $t + \Delta t$ can obtained as follows. Substituting $i = 1, 2, \ldots, L - 1$ in (14) which results in a tri-diagonal system of equations in unknown values of C. Using initial and boundary conditions, the system can be solved by Gauss elimination method Carnahan et al. (1969). Thus $C$ is known at all values of $y$ at time $t + \Delta t$. Then the known values of $C$ and applying the same procedure and using boundary conditions, similarly calculate $\theta$ and $u$ from (13) and (12). This procedure is continued to obtain the solution till desired time $t$. If $\lambda_i = M = m = K = Gc = S = Sc = Du = Sr = R = N = 0$, and $Gr = 1$, the results of Soundalgekar et al. (2004) are gotten.

Also, at $0 = m = K = Gc = S = Sc = Du = Sr = R = N = 0$, and $Gr = -1$, $\gamma = -1$, the results of Aruna Kumari et al. (2012) are obtained.

4.1 Velocity profiles

Figures 1 to 15 represent the velocity profiles with varying parameters respectively.

Figure 1 depicts the effect of Prandtl number on the velocity. It is observed that, the velocity decreases with increasing Prandtl number. Influence of Hartmann number $M$ on the velocity is presented in figure 2. It is found that, the velocity decreases with the increase in magnetic parameter. Figure 3 shows variation of Schmidt number on the velocity profile. It is noted that, the velocity decreases with increase in Schmidt number. Dufour number on the velocity profile is depicted in figure 4. It is observed that, the velocity decreases with increasing Dufour number parameter. Figure 5 illustrates different values of constant $\eta$ on the velocity. It is found that, the velocity increases with the increase of the constant. Effect of Jeffery parameter on the velocity is presented in figure 6. It is clear that, the velocity increases with increase in Jeffery parameter. Figure 7 depicts that with the increase in heat generation, the velocity of the fluid increases. Influence of suction parameter on the velocity is demonstrated in figure 8. It is seen that, the velocity is higher with due to an increase in suction parameter. Figure 9 shows different values of thermal Grashof number on the velocity, it is noted that, the velocity rises with increasing thermal Grashof number. In figure 10, the effect of mass Grashof number on the velocity is shown. It is observed that, the velocity increases with increase in mass Grashof number. The influence of Soret number on the velocity is given in figure 11. It is noticed that, the velocity rises with an increase in Sr. Figure 12 displays that for an increase in chemical reaction parameter, the velocity rises. Figure 13 illustrates the variation of radiation parameter on the velocity. It is shown that, the velocity rises with increase of radiation parameter. In figure 14, it is observed that, the velocity increases with
for different values of time. Figure 15 shows the effect of permeability parameter on the velocity. It is clear that, the velocity falls with increase in $K$.

Figure 1. Velocity profiles for different values of $Pr$.

Figure 2. Velocity profiles for different values of $M$. 
Figure 3. Velocity profiles for different values of Sc.

Figure 4. Velocity profiles for different values of Du.
Figure 5. Velocity profiles for different values of $\eta$.

Figure 6. Velocity profiles for different values of $\lambda_1$. 
Figure 7. Velocity profiles for different values of $S$.

Figure 8. Velocity profiles for different values of $\gamma$. 
Figure 9. Velocity profiles for different values of Gr.

Figure 10. Velocity profiles for different values of Gc.
Figure 11. Velocity profiles for different values of Sr.

Figure 12. Velocity profiles for different values of N.
Figure 13. Velocity profiles for different values of R.

Figure 14. Velocity profiles for different values of t.
Figure 15. Velocity profiles for different values of K.

4.2 Temperature profiles

Figures 16 to 22 illustrate the temperature profiles.

In figure 16, the influence of Prandtl number on the temperature is demonstrated. It is seen that, the temperature decreases when the Prandtl number is increased. Figure 17 represents effect of heat sink on the temperature. It is depicted that, the temperature increases with increase in heat generation. Variation of suction parameter on the temperature is given in figure 18. It is observed that, the temperature decreases with increase in the suction parameter. Figure 19 shows the effect of constant $\eta$ on the temperature. It is noted that, the temperature rises when the constant is higher. The effect of Dufour number on the temperature is shown in figure 20, it is obvious that the velocity increases due to rise in Du. In figure 21, it is presented that, the temperature increases with increasing time. Figure 22 depicts the variation of temperature for different values of radiation parameter, it is clear that the temperature increases with increase in R.
Figure 16. Temperature profiles for different values of Pr.

Figure 17. Temperature profiles for different values of S.
Figure 18. Temperature profiles for different values of $\gamma$.

Figure 19. Temperature profiles for different values of $\eta$. 
Figure 20. Temperature profiles for different values of Du.

Figure 21. Temperature profiles for different values of t.
4.3 Concentration profiles

Figures 23 to 27 show the concentration profiles. Effect of Schmidt number on the concentration is presented in figure 23. It is noted that, the concentration is lower due to increasing Schmidt number. In figure 24, the influence of suction parameter number on the concentration is shown. It is demonstrated that, the concentration is lower as the suction number is increased. Figure 25 displays the variation of chemical reaction parameter on the concentration. It is seen that, the concentration deceases with decreasing chemical reaction parameter. In figure 26, It is observed that the concentration rises with an increase in the time. Figure 27 depicts the variation of Soret number on the concentration. It is clear that, the concentration increases with increase in Soret number.
Figure 23. Concentration profiles for different values of Sc.

Figure 24. Concentration profiles for different values of Sc.
Figure 25. Concentration profiles for different values of $\gamma$.

Figure 26. Concentration profiles for different values of $t$. 
5.0 CONCLUSIONS

The present numerical study has been carried out for variable thermal conductivity on Jeffery fluid past a vertical porous plate with heat and mass fluxes. An implicit finite difference method of Crank-Nicolson type is used to solve the equations governing the flow. The following conclusions were drawn from the study:

1. The velocity becomes higher when thermal Grashof number, solutal Grashof number, Dufour number, heat Generation, Soret number, radiation, time and Jeffery parameter is increased. Also, decreases for Prandtl number, magnetic parameter, Schmidt number, permeability parameter, chemical reaction and suction lead to a sharp fall in the boundary layer.

2. The temperature profile increases in the presence of heat generation, radiation parameter, Dufour number, and time but reduces for increased values of Prandtl number and suction.

3. The concentration rises with increasing Soret number and time, and decreases with increasing values of Schmidt number Sc, chemical reaction parameter and suction.

4. An excellent agreement was seen with previous literatures when the results were compared.

References


